Chapter 7
Methods to Evaluate the Seismic Stability of Structures

7-1. Introduction

Structures must be evaluated with respect to sliding and rotation to ensure that they remain stable during an earthquake. Sliding stability of CHS under earthquake loading is evaluated using the limit equilibrium method (seismic coefficient) and permanent sliding displacement approaches (EM 1110-2-6050). Rotational stability of CHS under earthquake loading is evaluated using the energy-based formulation and the limit equilibrium method (EM 1110-2-6050). In addition to these methods, a new method based on rocking spectrum is introduced for assessment of rotational stability after a tipping of the structure has been indicated. All of these stability methods assume rigid structural behavior. This assumption is reasonable for most massive hydraulic structures, because the period of a sliding or rocking structure is much longer than the vibration period of the flexural response of the structure. However, the effects of structure flexibility on sliding and rotation could be important for more flexible and less massive structures and should be investigated. The structure flexibility can significantly affect the earthquake demands, which are used to determine whether or not sliding or rotation would take place. Sliding or rocking of a structure during an earthquake may not lead to failure of the structure. For a sliding failure to occur, the sliding displacement of the structure must be of sufficient magnitude to impair lateral load carrying capacity or life safety protection (for example, uncontrolled release of water from a reservoir). For a rotational stability failure to occur, the ground motion energy imparted to the structure after tipping occurs must be sufficient to cause rotational instability, or otherwise impair lateral load carrying capacity and life safety protection. Since bearing pressures can increase significantly as the resultant moves towards the edge of the base during a rotational response to earthquake ground motions, the load carrying capacity of the structure can be impaired due to a foundation bearing failure.

7-2. Rigid Structure vs. Flexible Structure Behavior

While a rigid structure will be subjected to a maximum acceleration equal to the peak ground acceleration (PGA) during earthquake ground shaking, a flexible structure will experience an average acceleration that depends on vibration period of the structure and on characteristics of the earthquake ground motion. This is illustrated by the acceleration response spectrum in Figure 7-1. The figure represents the typical acceleration responses of single-degree-of-freedom (SDOF) systems on a rock or firm soil site. Although most structures are not SDOF, a similar relationship can be assumed for the first-mode acceleration response of multi-degree of vibration systems. From Figure 7-1 it can be seen that only very rigid structures, with vibration period close to zero seconds, can be expected to experience peak accelerations equal to the PGA. For structures with periods between 0.02 seconds and 1 second (the typical range for most concrete hydraulic structures)
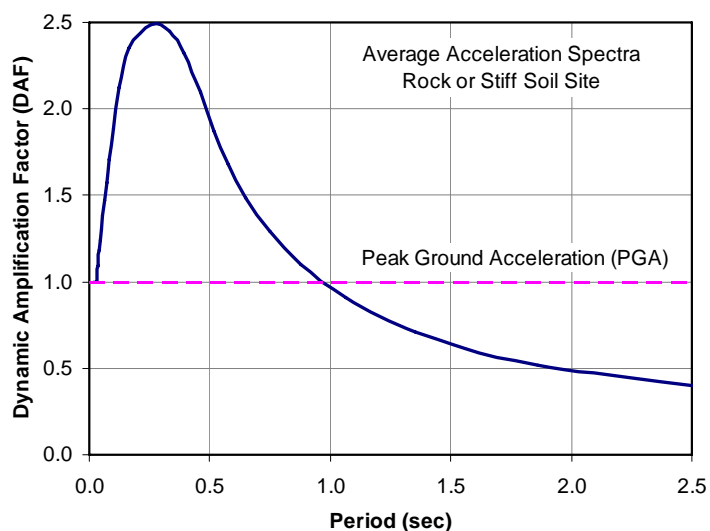

Figure 7-1. Dynamic Amplification Effects

the average structure acceleration will be greater than the PGA, with Dynamic Amplification Factors (DAF) as high as two to three.

7-3. Sliding Stability

a. Seismic coefficient method. In the limit equilibrium or seismic coefficient method, the sliding stability is expressed in terms of a prescribed factor of safety. A seismic coefficient, equal to 2/3 the peak ground acceleration divided by the acceleration of gravity (g), is used by the Corps to evaluate the potential for sliding. This coefficient when multiplied by the effective weight (structure weight + hydrodynamic added weight) provides the total lateral inertial force on the structure due to earthquake ground motions. The total lateral inertial force when added to static lateral forces, if any, provides the total driving force for the sliding stability analysis. The Maximum Design Earthquake (MDE) is considered an extreme load condition requiring a safety factor of 1.1 against sliding failure (Refer to EM 1110-2-2100 for stability requirements). A permanent sliding displacement analysis is required for structures that do not meet the required sliding factor of safety determined by the seismic coefficient method.

b. Permanent sliding displacement approach

(1) Upper bound estimate - rigid behavior. Sliding of a structure on its base will not occur until the total driving force exceeds the resisting force, or in other words when the sliding factor of safety is less than one. The total driving force can be due to static earth pressures, hydrostatic pressures, earthquake inertia forces, and earthquake induced hydrodynamic forces. Hydrodynamic forces are commonly determined by the Westergaard's added hydrodynamic mass (EM 1110-2-6051). The total mass of the system is therefore represented by the sum of the structure mass plus the hydrodynamic added mass. The static component of the driving force can easily be determined. The maximum inertia force for a rigid structure is a product of the total mass times the peak ground acceleration. The peak ground acceleration that will initiate sliding (i.e. when the driving force equals the resisting force) is defined as the critical acceleration. If the critical acceleration is greater than the peak ground acceleration of the design earthquake then the structure will not slide. Conversely, if the critical acceleration is less than the peak ground acceleration the structure will slide. An upper bound estimate of the permanent sliding displacement can be made using Newmark's rigid block analysis procedures (Newmark, 1965) or by methods developed by Richards and Elms (Richards and Elms, 1977). The Newmark procedure has been incorporated into the Corps program CSLIP. Newmark developed rigid block analysis procedures for rigid structures that slide in one direction only (dams, retaining walls, etc.) and for structure, which have the potential to slide equally in both directions (intake towers, lock monoliths, etc.). Newmark's sliding block analysis is demonstrated for a concrete gravity dam in Chopra and Zhang (1991), and the results from the Newmark analysis are compared to those obtained from a response history analysis. The potential for sliding, and the upper bound estimate of permanent sliding displacements can be reasonably determined using a Newmark-type sliding block analysis provided that the foundation sliding resistance is based on a best estimate (mean value) of the foundation shear strength, and that foundation shear strength parameters are adjusted for dynamic loading effects. Although the permanent sliding displacement is to be based on a mean shear strength value, permanent-sliding displacements should also be calculated using upper and lower bound estimates of foundation shear strength parameters.

(2) Upper bound estimate - flexible behavior. An approximate method based on rigid block analysis procedures (Chopra and Zhang, 1991) has been developed to estimate upper bound permanent displacements for flexible behavior. The analysis is similar to that used in the rigid

block analysis except that the sliding potential and estimate of upper bound displacements are based on the average peak structure acceleration rather than the peak ground acceleration. The average peak structure acceleration will generally be larger than the peak ground acceleration (see Figure 7-1) and therefore the upper bound permanent sliding displacement will be larger for a flexible structure than it is for a rigid structure. The average peak acceleration of the structure can be estimated by dividing the total first mode inertial force (base shear) obtained from a linear elastic response spectrum analysis by the total mass. Procedures for estimating average peak structure accelerations for flexible structures are provided in Chopra and Zhang, 1991.

c. Response history analysis procedures

(1) Linear time-history analysis – instantaneous factor of safety*.* The results of linear-elastic time-history analysis can be used to compute time-history or instantaneous sliding factor of safety along any desired sliding plane(s). The instantaneous factor of safety for the earthquake loading condition is obtained by combining the interface (i.e. sliding plane) force histories due to the earthquake loading with the interface forces due to the static usual loads plus the uplift. At each time step, the static and dynamic nodal forces are combined and then resolved into a resultant force having components normal and tangential to the sliding plane. The resisting forces are obtained from the normal component of the resultant force using the Mohr-Coulomb law, and the driving force is computed from vector summation of tangential components of the resultant force. The time-history of factor of safety is then obtained from the ratio of the resisting to driving forces at each time step. Figure 7-2 is an example of instantaneous factors of safety. The time-history starts at value equal to static factor of safety and then oscillates as the structure responds to the earthquake ground shaking. Under earthquake excitation, the stability is maintained and sliding does not occur if the factor of safety is greater than 1. However, a factor of safety of less than one indicates a transient sliding, which if repeated numerous times, could lead to excessive permanent displacement that could undermine safety of the structure. For example, Figure 7-1 shows that the factor of safety repeatedly falls below one, an indication that sliding of the structure could be expected. The magnitude of sliding displacement and its impact on the stability of the structure need to be evaluated by performing a nonlinear sliding displacement analysis discussed next.
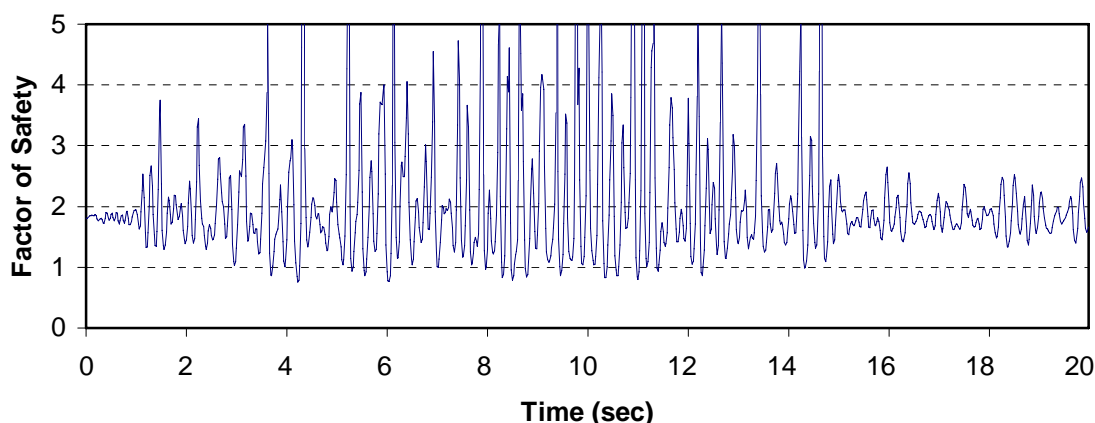

Figure 7-2. Time-history or instantaneous factors of safety

(2) Nonlinear time history analysis -- permanent sliding displacement. In nonlinear time-history analysis, governing equations of motion for the sliding structure are derived with respect to time and solved using step-by-step procedures (Chopra and Zhang 1991, Chavez and Fenves, 1993). A sliding structure is subjected to the ground acceleration plus the acceleration associated with the sliding displacement. If the sliding structure is assumed to be rigid, the governing equations involve dynamic equilibrium of inertia and static forces in the direction of sliding. Sliding is initiated when the acceleration reaches a critical or yield acceleration, i.e. a value at which the driving and resisting forces are equal; and the sliding ends when the sliding velocity becomes zero and the ground acceleration falls below the critical acceleration. If the sliding structure is flexible, two sets of governing equations will represent the sliding phase: 1) equations representing equilibrium of forces for the portion of the structure above the sliding plane, and 2) equations representing equilibrium for the entire sliding structure including all forces acting on the sliding plane. The structure's total permanent sliding displacement is then obtained by step-by-step solution of these coupled sets of equations. Alternatively, the nonlinear sliding behavior can be estimated using gap-friction elements along the sliding plane followed by a direct step-by-step integration of the equations of motion to obtain the total permanent sliding displacement.

7-4. Rotational Stability

a. General. A structure will tip about one edge of its base when earthquake plus static overturning moment (M_o) exceed the structure restoring moment capacity (M_r), or when the resultant of all forces falls outside the base. Depending on the magnitude of the peak ground acceleration, duration of main pulses, and slenderness of the structure, different rotational or rocking responses can be expected. As with sliding stability the inertia forces are likely to be larger for flexible structures than they are for rigid structures. Rotational or rocking responses to ground motions may include:

(1) No tipping because $M_o < M_r$

(2) Tipping or uplift because $M_o > M_r$, but no rocking due to insufficient ground motion energy

(3) Rocking response ($M_o > M_r$) that will eventually stop due to the energy loss during impact

(4) Rocking response that leads to rotational instability (extremely unlikely).

The likelihood of tipping can be determined by the following simple tipping potential evaluation. Even if tipping occurs, it is unlikely that it would result in rotational instability for the massive concrete hydraulic structures (Paragraph 7-4*d*). However, high bearing pressures can develop during tipping and rocking responses. A bearing failure evaluation is required to determine whether bearing pressures associated with the tipping and rocking responses could lead to foundation failure. Rocking spectrum and nonlinear time-history procedures are available to evaluate the potential for rotational instability (Paragraph 7-4d).

b. Tipping Potential Evaluation. Hydraulic structures subjected to large lateral forces produced by earthquakes may tip and start rocking when the resulting overturning moment becomes so large that the structure breaks contact with the ground. For a nearly rigid structure as shown in Figure 7-3, or for a flexible structure idealized as an equivalent single degree of freedom system, the tipping occurs when the overturning moment exceeds the resisting moment due to the weight of the structure. Note that in both cases it is assumed that the

structure is not bonded to the ground, but it may be keyed into the soil with no pulling resistance. This condition is expressed by:

$M_o > Mr$

$m\, S_a\, h > m\, g\, b$ or, $S_a > g\,(b/h)$ (7-1)

where:

M_o = overturning moment

M_r = resisting moment

S_a = spectral acceleration

g = gravitational acceleration

b = one-half width of the structure

h = distance from the base to the center of gravity

m = mass of the structure

This expression can also be used for hydraulic structures, except that the moments due to hydrostatic and hydrodynamic forces should be included and that the added hydrodynamic mass of water be also considered in determination of the structure's center of mass.
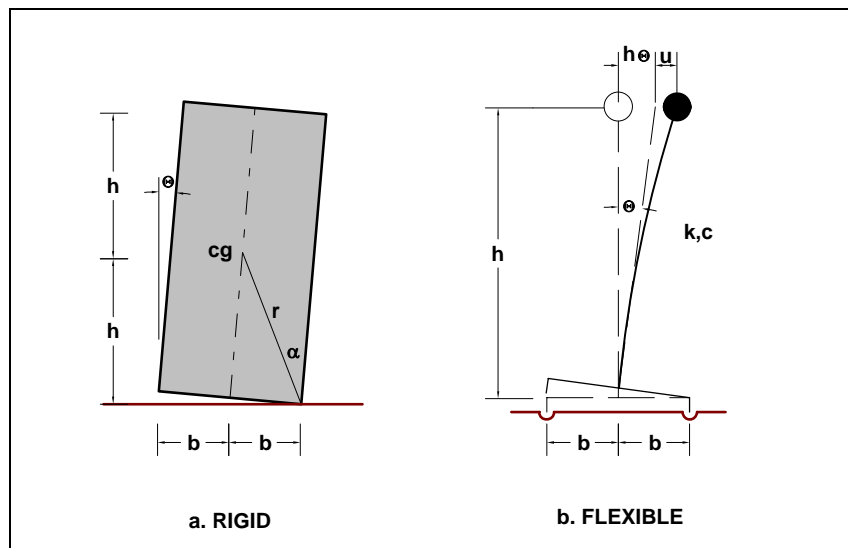

Figure 7-3. Rigid block and SDOF models for rigid and flexible structures

c. Energy Based-Rotational Stability Analysis. The structure will eventually overturn if the moment $M_o > M_r$ is applied and sustained. However, under earthquake excitation large overturning moments occur for only a fraction of second in each cycle, with intermediate opportunities to unload. Although rocking occurs, the structure may not become unstable rotationally if the energy loss during impact results in reduction of the angular velocity when the rotation reverses. By comparing the earthquake average energy input with the required average energy for overturning the structure, Housner provided the following approximate relationship as a criterion for the overturning stability of a rocking structure (Housner 1963):

$$\alpha = S_v \sqrt{\frac{mr}{gI_o}} \qquad (7\text{-}2)$$

where:

α = angle between the vertical and the line segment R as illustrated in Figure 7-2.

r = distance from the center of gravity to the corner about which rotation occurs.

I_o = mass moment of inertia about that corner.

S_v = spectral velocity of the earthquake ground motion.

Based on the average energy formulation used, this equation is interpreted as stating that for a given spectral velocity S_v, a block having an angle α given by Equation 7-2 will have approximately a 50 percent probability of being overturned (Housner 1963). For slender structures such as intake towers Equation 7-1 can be approximated by:

$$\alpha = \frac{S_V}{\sqrt{gr}} \qquad (7\text{-}3)$$

By combining Equations 7-1 and 7-3 and using the relationships among the spectral acceleration, velocity, and displacement, R. E. Scholl (ATC-10-01, 1984) found that consideration of one spectral parameter alone as the earthquake demand is not sufficient for evaluating overturning and suggested the following relationships:

$$S_d = b \quad \text{when} \quad S_d = g\frac{b}{h} \qquad (7\text{-}4)$$

These equations show that when S_a is just sufficient to cause tipping, the structure will start rocking, but its displacement approximated by spectral displacement S_d must reach the value b before it can overturn. These equations also demonstrate why larger structures such as buildings do not overturn during earthquakes, whereas smaller rigid blocks having the same aspect ratio are expected to overturn. This is because, in general, S_d is never large enough to tip over a building, but it can approach one-half the base width (i.e. b) of smaller rigid blocks such as tombstones. A better and more accurate procedure for evaluation of rocking response is the use of rocking spectra and nonlinear time-history method described next.

d. Time-history and rocking spectrum procedures

(1) Time history and rocking spectra can be used to estimate the uplift or overturning of hydraulic structures that tend to undergo rocking motion (Makris and Konstantinidis, 2001). There are distinct differences between a SDOF oscillator and the rocking motion of a rigid block, as shown in Figure 7-4. As such, an equivalent SDOF oscillator and standard displacement and acceleration response spectra should not be used to estimate rocking motion of structures. For example, the restoring mechanism of the SDOF oscillator originates from the elasticity of the structure, while the restoring mechanism of the rocking block from gravity. The SDOF oscillator

has a positive and finite stiffness, k, and energy is dissipated as the force-displacement curve forms closed loops. The rocking block, on the other hand, has infinite stiffness until the magnitude of the applied moment reaches the restoring moment, and once the block is rocking, its stiffness decreases and reaches zero when the of rotation of the block becomes equal to α (the block slenderness). The vibration frequency of a rigid block is not constant because it depends on the vibration amplitude (Housner 1963). The vibration frequency $p = (3g/4R)^{1/2}$ is a measure of the dynamic characteristic of the block. It depends on the size of the block, R, and the gravitational acceleration, g. This indicates that rocking response cycles of larger block is longer than the corresponding rocking response-cycles of the smaller block.

(2) Governing equations. The governing equations of rocking motion under horizontal ground acceleration are given by Yim et al. 1980, Makris and Roussos 2000, among others):

$$I_o \ddot{\theta}(t) + m\,g\,R\,sin(-\alpha - \theta) = -m\ddot{u}_g(t)R\,cos(-\alpha - \theta), \qquad for \quad \theta < 0 \quad (7\text{-}5)$$

$$I_o \ddot{\theta}(t) + m\,g\,R\,sin(\alpha - \theta) = -m\ddot{u}_g(t)R\,cos(\alpha - \theta), \qquad for \quad \theta > 0 \quad (7\text{-}6)$$

which in its compact form can be expressed as:

$$\ddot{\theta}(t) = -p^2\left\{ sin[\alpha\,sgn[\theta(t)] - \theta(t)] + \frac{\ddot{u}_g}{g}cos[\alpha\,sgn[\theta(t)] - \theta(t)]\right\} \qquad (7\text{-}7)$$

where for rectangular blocks;

$\alpha \quad = \tan^{-1}(b/h)$

$I_o \quad = (4/3)\,m\,R^2$

$P \quad = (3g/4R)^{1/2}$

a) Single-degree-of-freedom Oscillator

$\omega = \sqrt{\dfrac{k}{m}}$

$\xi = \dfrac{c}{2m\omega}$

c) Free-standing rocking block

$p = \sqrt{\dfrac{3g}{4R}}$

$a = \boldsymbol{tan}^{-1}\left(\dfrac{b}{h}\right)$

b) SDOF force-displacement diagram

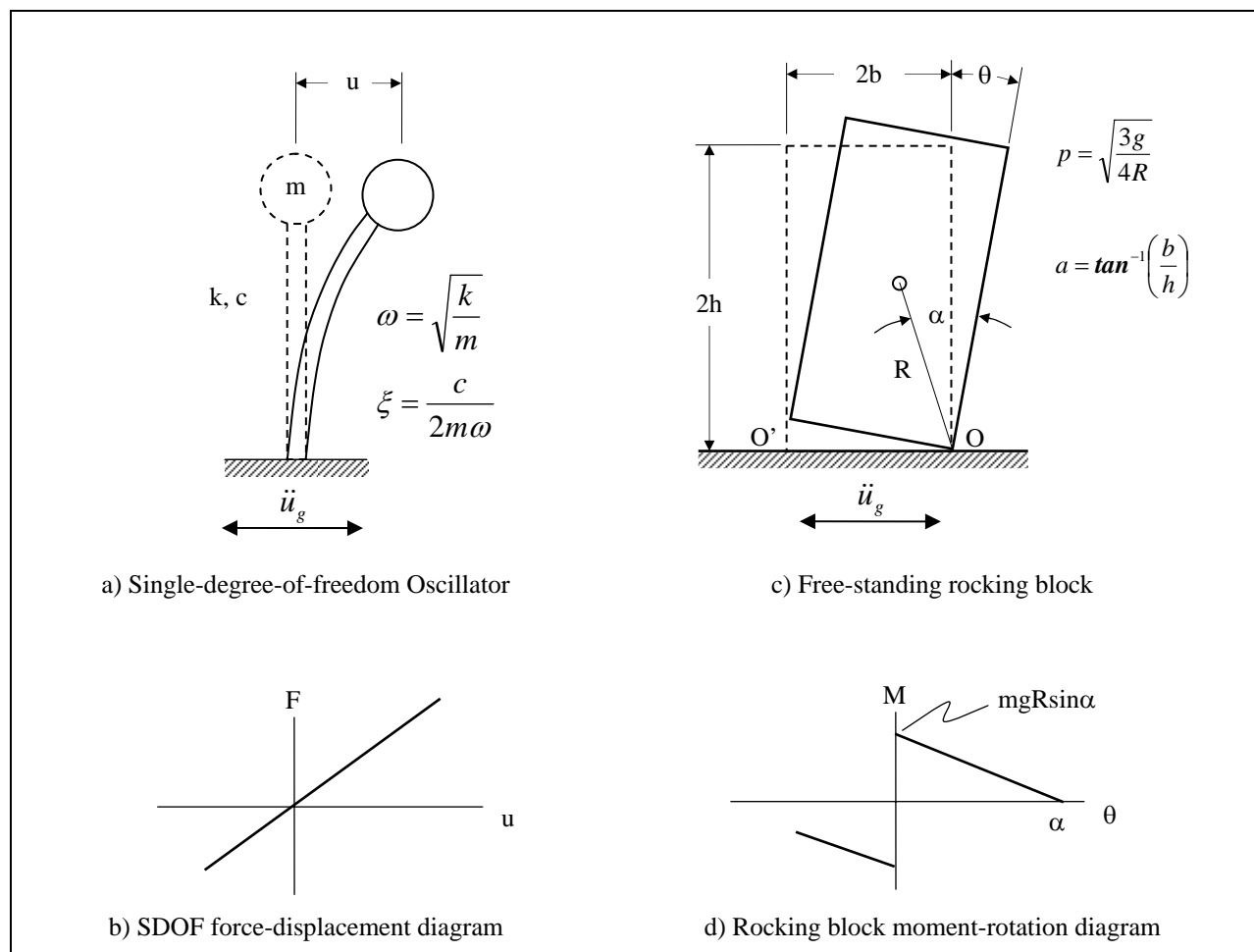d) Rocking block moment-rotation diagram

Figure 7-4. Comparison of a single-degree-of-freedom oscillator with a freestanding block in rocking motion (adopted from Makris and Konstantinidis, 2001)

(3) Time-history solution. The solution of Equation 7-7 is obtained by step-by-step numerical procedures. The rocking response quantity of interest include the block rotation, θ, and its angular velocity, $\dot{\theta}$. The resulting time-histories of θ and $\dot{\theta}$ will indicate how many impacts the block will experience and whether or not it will overturn (i.e. θ becomes greater than α).

(4) Rocking spectra. Same as the standard response spectra, one can generate rotational and angular velocity spectra (rocking spectra) as a function of the "period" $T=2\pi/p$ for different values of slenderness (damping), $\alpha = \tan^{-1}$ (b/h). This can be accomplished by solving Equation 7-7 for the maximum rotation of similar blocks of different sizes subjected to a given earthquake acceleration time history. This was done for similar blocks with $\alpha = 15°$ subjected to Pacoima Dam motion recorded during the 1971 San Fernando earthquake. The resulting rocking spectrum and the input acceleration record are shown in Figure 7-5. In the rocking spectrum, as $2\pi/p$ increases, the size of the block becomes larger. Larger values of the slenderness α correspond to larger amount of energy lost during impact. Figure 7-4 indicates that any block with slenderness α =15° that is small enough so that $2\pi/p<3.3$ sec (*or R<6.7* ft) will overturn when subjected to the Pacoima Dam record. Larger blocks with $2\pi/p>3.3$ sec *(or R>6.7* ft), will

uplift, but the maximum rotation is only a fraction of their slenderness. From this example, it should be obvious that rocking spectra provides a powerful and accurate tool for assessment of overturning potential of hydraulic structures. New research and development in this area are necessary to develop computation tools needed to make such assessments.
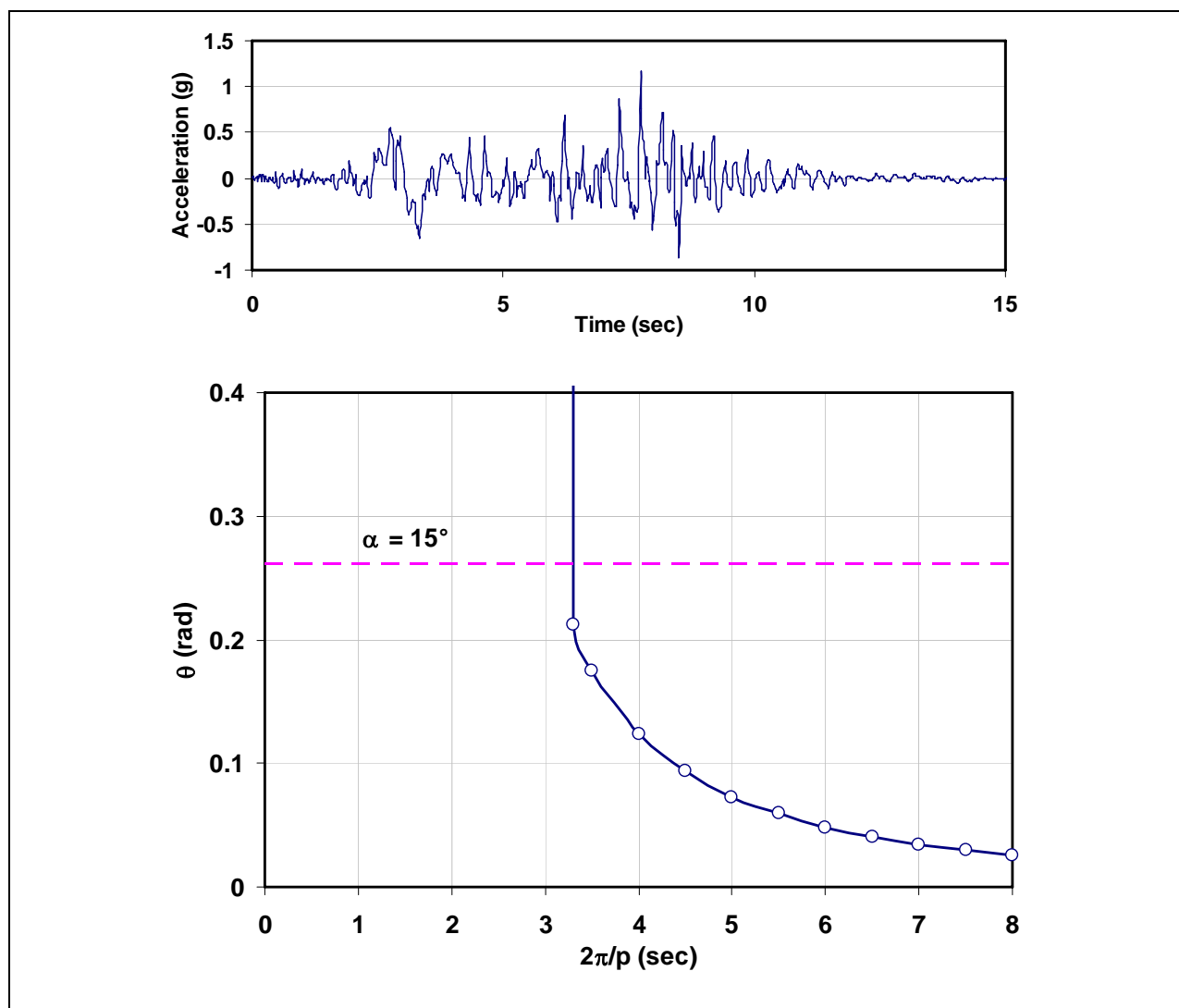

Figure 7-5. Pacoima Dam motion recorded during the 1971 San Fernando earthquake (top) and rocking spectrum of similar blocks with α = 15° (bottom).

7-5. Mandatory Requirements

   a. Performance requirements for stability shall be in accordance with EM 1110-2-2100, Stability Analysis of Concrete Structures.

   b. Seismic stability evaluation other than seismic coefficient method shall be in accordance with procedures discussed in this chapter.